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Ground State Wave Function of ^{9}Be using Resonating Group Method along with Complex Generator Coordinate Technique

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ABSTRACT: High energy elastic electron scattering allows the nucleus to be examined; it makes transitions to higher levels, which are higher than those levels that are reached by most of the other methods. From these studies values of transition matrix elements, angular momentum and parity are also found. This paper carries out the formulations of the cluster model wave function for ^{9}Be ground state nucleus using Resonating Group Method(RGM) along with the Complex Generator Coordinate Technique(CGCT) in the form of antisymmetrized product of single particle wave function. This wave function is then normalized, which can further be used to calculate the structural properties of the nucleus.

Key words: Cluster, Wave function, Antisymmeterization, Spin, Parity

I. INTRODUCTION

High energy elastic electron scattering is a powerful tool for studying the geometrical details of nuclear structure. The studies provide information on static distribution of charge and magnetization of nucleus. One can learn the details of the spatial distribution of transition charge and current density, which is a rich and unique information of the structure of nuclei.

These theoretical analyses of the electron scattering from nucleus have been carried out using wave functions corresponding to various models. The nuclear shell model, independent particle model has been used in number of cases. It was shown that these wave functions also give a good reproduction of the experimental data. But these correlations led to a 5 to 7 percent increase in the rms radii. Many other models have been also used such as deformed oscillator model, molecular orbital mode, independent particle, shell model and projected Hartree-Fock model. But they have not improved the situation substantially to fit the elastic scattering data.

A somewhat different approach has been used here. It is called as Complex Generator Coordinate Technique (CGCT). This technique is based on the fact that the antisymmetrization procedure can be simplified very much if the wave function of the total scattering system can be expressed as an integral of antisymmetrized product of single particle wave functions. I have introduced here integral representation of wave function, with no cross terms like r_i. r_j, where r_i and r_j are the space coordinates of nucleon *i* and *j* respectively.

In this wave function we have considered spin-isospin function, relative motion function, centre of mass of the system and antisymmetrization of the particles.

II. WAVEFUNCTION FOR THE GROUND STATE OF ^{9}Be

The ground state wave function of ^{9}Be nucleus is written with definite spin and parity using cluster model. In ground state ^{9}Be nucleus consists of four protons and five neutrons. Since alpha clusters are tightly bound energetically, we have considered ^{9}Be nucleus as consisting of a core of two alpha clusters making ^{8}Be core in its ground state and third neutron cluster moving relative to the centre of mass of ^{8}Be as shown in figure.

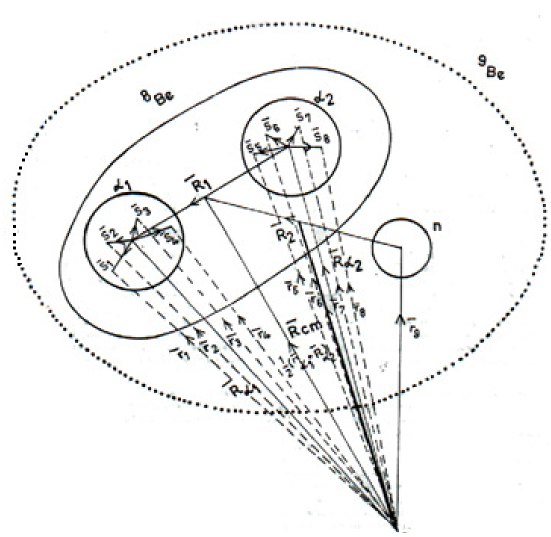

Fig.1

The spin-parity of ^{9}Be ground state are $J^{\pi} = \frac{3^-}{2}$ and those of ^{8}Be ground state are $J^{\pi} = 0^+$. The cluster model wave function for ^{9}Be can be written as:

$$\Phi_{\frac{3}{2},\frac{3}{2}} = \mathrm{A}\, [\phi_{00}\, (^8Be)\, \chi_{11}\, (\bar{R}_2)\, \xi\, (\sigma, \tau)]$$

$$= \mathrm{A}\, [\phi(\alpha_1)\, \phi(\alpha_2)\, \chi_{00}\, (\bar{R}_1) \chi_{11}\, (\bar{R}_2) Z\, (\bar{R}_{cm})\, \xi_9(\sigma, \tau)\,] \quad \ldots(1)$$

Where A is the antisymmetrization operator and Φ_{00} (^{8}Be) is the ground state wave function of ^{8}Be, χ_{00} is the relative motion wavefunction between two alpha clusters, χ_{00} is the relative motion wavefunction between two alpha and one neutron cluster, R_{cm} is the centre of mass function, $\xi_g\, (\sigma, \tau)$ is the spin-isospin function for nine particle system.

Now introducing an integral representation for $\Phi_{\frac{3}{2},\frac{3}{2}}$ which is the form of an antisymmetrized product of single particle wave functions :

$$\Phi_{\frac{3}{2},\frac{3}{2}} = \left(\frac{1}{4\pi^2}\right)^3 \mid \mathrm{J} \mid \int A\, \prod_{j=1}^{4} \prod_{K=5} \xi_{\alpha_1}\, \xi_{\alpha_2}\, \xi_9 \exp\left\{-\frac{\alpha}{2}\, (\bar{r}_j - i\bar{P})^2\right\}$$

$$\times\, \exp\left\{-\frac{\alpha}{2}\, (\bar{r}_K - i\bar{Q})^2\right\} \exp\left\{-\frac{\alpha}{2}\, (\bar{r}_9 + 4i\bar{P} + 4i\bar{Q})^2\right\}$$

$$\times\, \chi_{00}\left(\bar{R}_1''\right) \chi_{11}\left(\bar{R}_2''\right) \exp[-2\alpha\, \{(\bar{P} + \bar{Q})^2 - 2\bar{P}\,.\bar{Q}\} + \alpha R_1''^2$$

$$+ \frac{4}{9}\, \alpha\, R_2''^2 - 2i\alpha \bar{R}_1''\,.(\bar{P} - \bar{Q}) - \frac{4i\alpha}{9}\, \bar{R}_2''.(\bar{P} + \bar{Q}) - \frac{32}{9}\, i\alpha\, \bar{R}_2''.(\bar{P} + \bar{Q})$$

$$- 8\alpha\, (\bar{P} + \bar{Q})^2\,]\,.\, d\bar{P}\, d\bar{Q}\, d\bar{R}_1''\, d\bar{R}_2'' \quad \ldots(2)$$

The term having double prime denotes parameter coordinates. Antisymmetrizer operator does not act on parameter coordinates.

Making coordinate transformation as follows:

$$\bar{P} = \frac{1}{2\alpha}\, (\bar{J}'' + \bar{K}'')$$

$$\bar{Q} = -\frac{1}{2\alpha}\, (\bar{J}'' - \bar{K}'') \quad \ldots(3)$$

And using the Jacobian of transformation:

$$d\bar{P}\, d\bar{Q} = |J^*|\, d\bar{J}''\, d\bar{K}'' \quad \ldots(4)$$

Substituting these equations in eqn. (2) and evaluating |J| from the determinant of the order of 15×15 and |J*| from the determinant of the order 6 × 6 we get,

$$\mid \mathrm{J} \mid = (16\mathrm{a}^2)^3$$

$$|J^*| = \left(\frac{1}{2\alpha^2}\right)^3 \quad \ldots(5)$$

Finally the equation of wave function for ^{9}Be ground state is written as:

$$\Phi_{\frac{3}{2},\frac{3}{2}} = \left(\frac{2}{\pi^2}\right)^3 \int A\, \prod_{j=1}^{4} \prod_{K=5}^{8} \xi_{\alpha_1}\, \xi_{\alpha_2}\, \xi_9 \exp\left[-\frac{\alpha}{2}\, (\bar{r}_j - \frac{1}{2\alpha}\, (\bar{J}'' + \bar{K}'')^2\right]$$

$$\times \exp.\left[-\frac{\alpha}{2}\, \left\{(\bar{r}_k + \frac{1}{2\alpha}\, (\bar{J}'' - \bar{K}'')\right\}^2\right] \exp\left[-\frac{\alpha}{2}\, \left\{\bar{r}_9 + \frac{4i\bar{K}''}{2\alpha}\right\}^2\right]$$

$$\chi_{00}\left(\bar{R}_1''\right) \chi_{11}\, (\bar{\mathrm{R}}_2'') \exp\left[\frac{1}{\alpha}\left\{\left(\alpha\bar{R}_1'' - i\bar{J}''\right) + \left(\frac{2}{3}\alpha\bar{R}_2'' - 3i\bar{K}''\right)\right\}\right] d\bar{R}_1''\, d\bar{R}_2''\, d\bar{J}''\, d\bar{K}'' \quad \ldots(6)$$

The normalization constant is calculated by the expression $\langle \phi_{\frac{3}{2},\frac{3}{2}} | \phi_{\frac{3}{2},\frac{3}{2}} \rangle$.

III. CONCLUSION

In this paper by the use of nuclear cluster model, wave function with definite parity and angular momentum is formulated for ^{9}Be nucleus in the ground state. The CGCT allows the transformation of the cluster model wave function written in terms of cluster coordinates to antisymmetrized product of single particle wave function written in terms of single particle coordinates, the centre of mass coordinate and a number of parameter and generator coordinates. We have already constructed the wave function of ^{5}Be nucleus using same method and have calculated rms radius, quadrupole moment and charge form factor. The calculated normalized wave function can be further used to calculate rms radius, quadrupole moment, charge form factor etc. of the ground state of ^{9}Be nucleus which is of great interest.

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